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PROOF OF FERMAT'S THEOREM

To Professor W. W. Beman,
Compliments of
M. A. McGinn

PROOF OF FERMAT'S THEOREM

AND

McGINNIS' THEOREM OF DERIVATIVE EQUATIONS IN AN ABSOLUTE PROOF OF FERMAT'S THEOREM; REDUCTION OF THE GENERAL EQUATION OF THE FIFTH DEGREE TO AN EQUATION OF THE FOURTH DEGREE; AND SUPPLEMENTARY THEOREMS

BY

MICHAEL ANGELO McGINNIS

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Dedication

Born upon what was once French soil, this little volume is gratefully dedicated to the memory of the immortal *Fermat*, whose worthy achievements in the field of mathematical research have added much to the knowledge and happiness of the human race. He bequeathed to posterity a new-born star that will continue to shine when the last sun goes down to rise no more. While Scientific France reveres and does honor to his memory, all nations and all peoples claim him; for such men belong to no nation and to no clime. They are the stars and suns that light the world, revealing the hidden links of *Truth* in that endless chain which binds man to his Creator, and each newly discovered link is but a step upward, drawing him closer and nearer to his God.

Scientific Truths are immortal things emanating from Divinity. They are thought vibrations proceeding from the *First* — the *Only* — *Supremely-Perfect Cause* of all things that be, and can not be received and reflected by unsound and impure minds. *Fermat* was truly one of God's most perfect, purest souls, toiling in the field of mathematics, in which he saw the laws of God shimmering in the sunbeam, vibrating upon the bosom of ocean, and eddying in every gust of wind.

All praise to the name and memory of *Pierre Fermat.*

THE AUTHOR.

PREFACE

THIS little volume contains my proof of *Fermat's Theorem,* — "The sum of no two powers except squares is itself a power of the same degree."

My attention was first called to the existence of this now celebrated problem (judging from the number of proposed solutions arriving almost daily in Göttingen) by reading a synopsis of a lecture on "Mathematical Research" delivered in New York by Professor G. A. Miller of Urbana, Illinois, and published in the "Literary Digest" June 29, last. The copy of the "Digest" was given to me by Mr. John F. Dwyer of Jefferson City. I am, therefore, indebted to Professor Miller for his lecture, and to Mr. Dwyer for the copy of the "Literary Digest."

The proof speaks for itself, and presents to my mind many new and important ideas, one of which I shall take the liberty to mention. It is this: If the dimensions of or in space are not infinite, then, the number must be limited to *three;* and the assumed *fourth* dimension is purely imaginary from a mathematical standpoint, and can not exist either in Time or Space, — *but may exist independently of Time and Space!* (In the analysis and proof of this proposition will be found the link that connects pure mathematical reasoning with that of abstract philosophy, thus giving to the latter a true place and meaning in Science.)

The "Theorem of Derivative Equations," in its application, furnishes an absolute proof that the sum of the *n*th powers of two numbers can not be an nth power of a third number when $n > 2$.

I have not had the time nor convenience to test the "Theorem of Derivative Equations" in the Equations of Curves; but I am satisfied it will add new light and meaning to the theory of derivatives found in the *Calculus*.

The proofs of Theorems I and II I believe to be well established; and in the linking of the proofs (**38–39**), additional proof is given to the proof of the proposed theorems of the writer in an absolute and final proof of *Fermat's Theorem*.

All theorems and remarks in the supplementary addition form no part of the proof of *Fermat's Theorem*. These theorems are the result of the author's successful effort (after seven years of toil) to solve the general equation of the *third degree* without recourse to "Cardan's ingenious device." A true general solution of the Cubic is the Key to general solutions of all degrees.

In 1900, I succeeded in reducing the general equation of the *sixth degree* to an equation of the *fifth degree*, and discovered that all *even* degree equations can be reduced to an equation *one degree lower*. But, I found that *odd* degree equations would not yield to the same method of attack.

The general equation of the nth degree, when n is even, has $(n-1)$ algebraically symmetrical functions; and, when n is odd, it has n such functions. An algebraically symmetrical function is a function of the roots of an equation that can be algebraically expressed in terms of the coefficients of the equation. Thus: If a, b, c, d are the roots of the biquadratic, then, we have: $ab+cd$, $ac+bd$, and $ad+bc$. These three functions will form a *cubic equation*. The algebraically symmetrical functions of the *sixth* are given in the latter part of this volume.

The reduction of odd degree equations depends upon the proof of Theorems I, II, and III in the supplementary addition (**42**). The proofs of Theorems I and III are so extensive that I am unable to present them here. (If I live, and opportunity offers, they shall be published.)

Due credit must be given to L. C. Hjorth & Sons, who deem this little volume of sufficient scientific value to finance the publication of the same for the sole benefit of the author and in the interest of science, — and thus expressing their love for the advancement of science, and their lack of purely selfish motives.

To Rev. Henry A. Geisert of Jefferson City is due the credit of assisting the author in many ways, and interesting himself in the speedy publication of the work.

And while we thus feel grateful to those who lend assistance in whatever form, we must needs remember that, although the problem was promulgated by a Frenchman, it finds its proof, if to be found at all, in the generosity of the great German thinker and scholar, *Dr. Paul Wolfskehl*, who, by his munificent gift, has invited the scholars of the civilized world to seek the proof. There can be but little doubt, if indeed any, but that Dr. Wolfskehl saw that many new and valuable ideas would find existence in the investigation of the proof of Fermat's Theorem that would prove to be more valuable to humanity in the advancement of science than what might be expected from the mere proof itself. He shall always be considered by those who seek truth for its own sake as one of the world's most worthy benefactors — one who loved his fellow man, and *truth, better than gold*.

I study the science of mathematics for the truths it contains. It makes man better, nobler. It makes him love truth, and despise falsehood, and hate injustice. And knowing that "in science there are no social, religious, or international boundary lines," — I turn to Germany, the land of thinkers and scholars, for a verdict through her Royal Academy of Scientific Researches of Göttingen. I shall be satisfied if it be found that I discovered *but one* new and valuable truth in the boundless field of mathematical research in which I feel naught — but a weak, devoted student.

MICHAEL ANGELO McGINNIS.

KANSAS CITY, MO., U.S.A.
January, 1913.

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PROOF OF FERMAT'S THEOREM

1. Fermat's Theorem.

The Sum of No Two Powers except Squares is Itself a Power of the Same Degree.

2. The proof of "Fermat's Theorem" depends upon the proof of the following propositions and theorems :

(*a*) To divide any number into two such parts that the sum of the squares of the parts will equal the square of a third number.

(*b*) To divide a given straight line into two such parts that the sum of the squares described upon the parts will equal the square described upon a third line.

(*c*) From the sum of the solid contents of two cubes to construct a third and perfect cube.

(*d*) To construct a triangle two of whose sides are known, and its third side the *n*th root of the sum of the *n*th powers of its two given sides.

(*e*) **Theorem I.** If the quantities α and β are increasing functions such that $\alpha = 1, 2, 3, 4, \cdots$ for all numbers, and $\beta = 2, 3, 4, 5, \cdots$ for all numbers except 1, and $\beta > \alpha$, at all times, when combined with α, then if α remains permanent while β assumes all possible values of the *n*th degree greater than α, the *n*th root of the sum of $\alpha + \beta^n$ is incommensurable when $n > 1$.

(*f*) **Theorem II.** If the quantities α and β represent positive, integral, or fractional numbers, then the *n*th root of the sum of $\alpha^n + \beta^n$ is incommensurable when $n > 2$.

(*g*) **McGinnis' Theorem of Derivative Equations.** The *general equation of the nth degree* has *n* derivative equations of the $(n-1)$th degree, and each derivative equation contains *at least one root* of the equation from which it is derived.

(*h*) **Theorem III.** The $(n-1)$th root of n is **incommensurable** when $n > 2$.

(*i*) **Theorem IV.** The nth root of the sum of $\alpha^n + \beta^{an}$ is incommensurable when $\beta > \alpha$, and $n > 1$. And the (an)th root of the sum of $\alpha^n + \beta^{an}$ is incommensurable when $\beta > \alpha$, and $n > 1$.

3. To Prove (*a*) – Art. 2.

As the sum of the squares of two numbers is to be considered, we must arrange or combine all numbers in pairs taken two at a time. That is: the square of every conceivable number must be added to the square of every other conceivable number except itself.

(1) Assume that the quantities α and β are increasing functions such that

(2) $\alpha = 1, 2, 3, 4, 5, \cdots$, for every conceivable number;

(3) and $\beta = 2, 3, 4, 5, 6, \cdots$, for every conceivable number except 1.

We will then have for the sum of the squares of any two numbers (except like numbers), the following formulæ:

$$\text{(4) The sum of } \alpha^2 + \beta^2 = \begin{cases} 1 + 2^2,\ 1 + 3^2,\ 1 + 4^2,\ \cdots,\ \text{to } 1 + \beta^2{}_m, \\ 2^2 + 3^2,\ 2^2 + 4^2,\ 2^2 + 5^2,\ \cdots,\ \text{to } 2^2 + \beta^2{}_m, \\ 3^2 + 4^2,\ 3^2 + 5^2,\ 3^2 + 6^2,\ \cdots,\ \text{to } 3^2 + \beta^2{}_m, \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{cases}$$

and so on, for the sum of the squares of any two numbers, in which the square of every conceivable number is added to the square of every other conceivable number except itself. (The sum of two like powers as $\alpha^2 + \alpha^2$ will be considered later on.)

Let us assume that it is possible for the following equality to exist, viz.:

(5) $$\alpha^2 + \beta^2 = \gamma^2.$$

(6) Assume that $\beta > \alpha$, at all times, when combined.

(7) If the sum of $\alpha^2 + \beta^2$ is a power of the second degree, it will be possible to construct a right triangle whose sides will be α, β, and γ, —γ being the square root of the sum of $\alpha^2 + \beta^2$.

(8) As $\beta > \alpha$, $\gamma > \beta$. (The square root of the sum of the squares of two numbers or quantities is greater than the greater number.)

(9) $\alpha + \beta > \gamma$. (The sum of two sides of a triangle is greater than the third side.) (See any geometry for proof.)

(10) Assume that $\gamma - \beta = \Delta\beta$. Then $\Delta\beta + \beta = \gamma$. Let $\Delta\beta = x$. Then, $x + \beta = \gamma$.

We now have the following equations:

(11) $$(x + \beta)^2 = \gamma^2 = \alpha^2 + \beta^2 = (\Delta\beta + \beta)^2.$$

Expanding $(x + \beta)^2$ we have

(12) $$x^2 + 2\,\beta x + \beta^2 = \gamma^2 = (\Delta\beta + \beta)^2 = \alpha^2 + \beta^2.$$

By transposition of $\alpha^2 + \beta^2$ for γ^2 we have

(13) $$x^2 + 2\,\beta x - \alpha^2 = 0.$$

Dividing the roots of (13) by β, we have, writing y for the new value of x, being $\frac{x}{\beta} = \frac{\Delta\beta}{\beta} = \Delta = y$,

(14) $$y^2 + 2\,y - \frac{\alpha^2}{\beta^2} = 0,$$

(15) $$y + 1 = \pm\left[\frac{\alpha^2}{\beta^2} + 1\right]^{\frac{1}{2}}.$$

Transposing $\frac{\alpha^2}{\beta^2}$ in (14) to the right of the sign of equality and adding 1 to both sides of the equation and taking the square root of both members.

As a general rule the sum of $\frac{\alpha^2}{\beta^2} + 1$ is not a power of the second degree; and equation (14), having a perfect power for its absolute term, and its sign *minus*, and all the other coefficients of the equation whole numbers, and their sign plus, and the coefficient of the highest power unity, can not have a rational fraction for one of its roots, unless when the sum of $\alpha^2 + \beta^2$ is a perfect square. This will occur in all cases where the values of α and β are amenable to Law I. Thus: When $\alpha = 3$, or 5, and $\beta = 4$ or 12, $\alpha^2 + \beta^2 = \gamma^2 = 3^2 + 4^2 = 5^2$; $5^2 + 12^2 = 13^2$.

The numbers 3, 4, and 5, and 5, 12, and 13 represent in each case the three sides of a right triangle.

(16) We may now write the general formula for all such numbers — the sum of the squares of which are powers of the same degree:

$$(n\,3)^2 + (n\,4)^2 = (n\,5)^2; \text{ and } (n\,5)^2 + (n\,12)^2 = (n\,13)^2.$$

n may represent any positive or negative number, *integral* or *fractional*.

(17) Any number that is exactly divisible by 7, or 17, can be separated into whole numbers, the sum of the squares of which will be a power of the second degree. Thus: To divide 14 and 34 into two such parts that the sum of the squares of the parts will be a perfect square.

SOLUTION: $14 \div 7 = 2 = n$ in formula. $\therefore 2 \times 3 = 6$, and $2 \times 4 = 8$. $6^2 + 8^2 = 10^2$. $34 \div 17 = 2 = n$. $\therefore 2 \times 5 = 10$, and $2 \times 12 = 24$.
$10^2 + 24^2 = 26^2$.

If the number to be divided is not an exact multiple of 7 or 17 (or the sum of any two numbers, the sum of the squares of which is a power of the second degree), we will obtain fractional results.

LAW I. When the square of the sum of two numbers is diminished by twice their product and there remains a perfect square, then, the sum of the squares of such numbers will be a power of the second degree. And, geometrically: If the square described upon the sum of two lines is diminished by twice the rectangle of the lines and there remains a perfect square, then, the sum of the squares described upon such lines will be equal to the square described upon a third line.

In all other cases imperfect powers of the second degree are the results obtained.

It is therefore possible to construct the proposed triangle when the sum of the numbers or lines complies with Law I. It is therefore possible, in special cases, for the following equality to exist: $\alpha^2 + \beta^2 = \gamma^2$.

Therefore, Fermat's Theorem holds good for powers of the second degree.

REMARK. Further discussion of the sum of $\alpha^2 + \beta^2$ will be given in the *Theorem of Derivative Equations*.

13. To prove the proposed theorem of Fermat for the sum of any two powers of the third degree, we build the following formulæ:

$$\text{The sum of } \alpha^3 + \beta^3 = \begin{cases} 1 + 2^3, \ 1 + 3^3, \ 1 + 4^3, \ \cdots \ 1 + \beta^3_m, \\ 2^3 + 3^3, \ 2^3 + 4^3, \ 2^3 + 5^3, \ \cdots \ 2^3 + \beta^3_m, \\ 3^3 + 4^3, \ 3^3 + 5^3, \ 3^3 + 6^3, \ \cdots \ 3^3 + \beta^3_m, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{cases}$$

and so on, ad infinitum, for the sum of any two powers of the third degree, in which the cube of every conceivable number is added to the *cube* of every other conceivable number, except itself.

14. The sum of any two like powers of the nth degree cannot be equal to the nth power of a third number.

(1) The proof of the above proposition can be easily demonstrated as follows:

(2) Assume that $\alpha^3 = 1, 2^3, 3^3, 4^3, 5^3, \cdots$ for the cubes of all numbers.

(3) $\alpha^3 + \alpha^3 = 2\,\alpha^3$.

Then the cube root of the sum of $\alpha^3 + \alpha^3 =$ the cube root of $2\,\alpha^3$, which is $\alpha\sqrt[3]{2}$. In this case, $n = 3$; when $n = 4$, then, we have $\alpha\sqrt[4]{2}$; when $n = 5$, we have $\alpha\sqrt[5]{2}$, and so on, ad infinitum. We can then write the general expression for the nth root of the sum of any two like powers, as follows:

(4) $(\alpha^n + \alpha^n)^{\frac{1}{n}} = \alpha(2)^{\frac{1}{n}}$. And as the nth root of 2 is incommensurable for all values of $n > 2$, it follows that the sum of any two like powers can not be a power of the same degree when $n > 2$.

15. The sum of two cubes is often equal to the square of a third number. Thus, $1 + 2^3 = 3^2$, and $4^3 + 8^3 = 24^2$. $4 + 8$ is but a multiple of $1 + 2$. When it is found that the sum of two cubes is equal to the square of a third number, then, we may write a general formula that will embrace all such numbers, viz.:

$$(n^2\,1)^3 + (n^2\,2)^3 = (n^3 \times 3)^2 \text{ for } (1 + 2^3 = 3^2).$$

In general,

$$(n^2\alpha)^3 + (n^2\beta)^3 = (n^3 \times d)^2$$

in which $\alpha^3 + \beta^3 = d^2$. $n = 1, 2, 3, \cdots$ for all numbers.

16. We now lay down the law for the sum of two cubes which will equal a perfect square:

LAW II. When the cube of the sum of any two numbers is diminished by three times their product into their sum, and there remains a perfect square, then, the sum of the cubes of such numbers will be a power of the second degree. Thus:

$$(\alpha+\beta)^3 - 3\,\alpha\beta(\alpha+\beta) = \gamma^2.$$

When such proves to be the case, $\alpha^3 + \beta^3 = \gamma^2$.

REMARK. The sum of three cubes is often equal to a fourth cube. Thus: $3^3 + 4^3 + 5^3 = 6^3$.

Often the sum of two cubes is equal to the sum of two other cubes. Thus: $9^3 + 10^3 = 12^3 + 1^3$; and $(12^3 + 1^3) \div 13 = 2^3 + 5^3$; and $(9^3 + 10^3) \div 19 = 3^3 + 4^3$.

But we can not find from the formula (13), the sum of two cubes which will give us a power of the third degree; and as the number of combinations is infinite, it is impossible to build an arithmetical solution that will prove in any way to be general in character.

16½. It is useless and a waste of time to carry on a discussion of the proof of "Fermat's Theorem" from a purely arithmetical standpoint. To establish the truth of "Fermat's Theorem," we must found, or build, a proof that is arithmetical, geometrical, and algebraic in character.

17. To Prove (*c*), Art. 2.

Let a and b represent the sides of two cubes the solid contents of which are, respectively, a^3 and b^3. We are to prove that the sum of

(I) $a^3 + b^3 = c^3$ (assumed to be a rational quantity or number).

Assume that the quantities a and b represent positive, integral, or fractional numbers; and that each represents the side of a cube. Then will the cube root of the sum of their cubes be a rational number or quantity which we designate by c?

If the sum of $a^3 + b^3$ be a power of the same degree, then it will be possible to construct a third cube whose side is assumed to be c from the sum of $a^3 + b^3$.