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Cyrill Stachniss

Robotic Mapping and Exploration

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For Janna and Maren

Series Editor's Foreword

By the dawn of the new millennium, robotics has undergone a major transformation in scope and dimensions. This expansion has been brought about by the maturity of the field and the advances in its related technologies. From a largely dominant industrial focus, robotics has been rapidly expanding into the challenges of the human world. The new generation of robots is expected to safely and dependably co-habitat with humans in homes, workplaces, and communities, providing support in services, entertainment, education, health-care, manufacturing, and assistance.

Beyond its impact on physical robots, the body of knowledge robotics has produced is revealing a much wider range of applications reaching across diverse research areas and scientific disciplines, such as: biomechanics, haptics, neurosciences, virtual simulation, animation, surgery, and sensor networks among others. In return, the challenges of the new emerging areas are proving an abundant source of stimulation and insights for the field of robotics. It is indeed at the intersection of disciplines that the most striking advances happen.

The goal of the series of Springer Tracts in Advanced Robotics (STAR) is to bring, in a timely fashion, the latest advances and developments in robotics on the basis of their significance and quality. It is our hope that the wider dissemination of research developments will stimulate more exchanges and collaborations among the research community and contribute to further advancement of this rapidly growing field.

The monograph written by Cyrill Stachniss is a contribution in the area of self-localization and mapping (SLAM) for autonomous robots, which has been receiving a great deal of attention by the research community in the latest few years. The contents expand the authors doctoral dissertation and are focused on the autonomous mapping learning problem. Solutions include uncertainty-driven exploration, active loop closing, coordination of multiple robots, learning and incorporating background knowledge, and dealing with dynamic environments. Results are accompanied by a rich set of experiments, revealing a promising outlook toward the application to a wide range of

mobile robots and field settings, such as search and rescue, transportation tasks, or automated vacuum cleaning.

Yet another STAR volume on SLAM, a very fine addition to the series!

Naples, Italy
February 2009

Bruno Siciliano
STAR Editor

Foreword

Simultaneous localization and mapping is a highly important and active area in mobile robotics. The ability to autonomously build maps is widely regarded as one of the fundamental preconditions for truly autonomous mobile robots. In the past, the SLAM has mostly been addressed as a state estimation problem and the incorporation of control into the map learning and localization process is a highly interesting research question. In this book by Cyrill Stachniss, the reader will find interesting and innovative solutions to the problem of incorporating control into the SLAM problem. I know Cyrill since over eight years and I still appreciate his enthusiasm in developing new ideas and getting things done. He has been working with a large number of different robots, participating in several public demonstrations, and has gained a lot of experience which can also be seen from his large number of papers presented at all major robotic conferences and in journals. His work covers a variety of different topics. He has acquired several project grants and received several awards. He furthermore is an associate editor of the *IEEE Transactions on Robotics*. It's safe to say that he is an expert in his field.

This book is a comprehensive introduction to state-of-the-art technology in robotic exploration and map building. The reader will find a series of solutions to challenging problems robots are faced with in the real world when they need to acquire a model of their surroundings. The book focuses on autonomy and thus the robot is not supposed to be joysticked though the world but should be able to decide about his actions on its own. I regard the ability to learn maps by making own decisions as a key competence for autonomous robots. Cyrill rigorously applies probabilistic and decision-theoretic concepts to systematically reducing the uncertainty in the belief of a robot about its environment and its pose in the environment.

The book contains impressively demonstrates the capabilities of the described solutions by showing results obtained from real robotic datasets. A further strength lies in the sound and thorough evaluation of all presented techniques going beyond the world of simulation. At this point, I would like to encourage the reader to follow Cyrill's example to take real

robots and data obtained with real robots to demonstrate that novel approaches work in reality. For readers not in possession of particular sensors or for comparison purposes, Cyrill and colleagues have created a Web site (<http://www.openslam.org/>) in which the community can share implementations of SLAM approaches and where the reader will find links to datasets to support future research.

Freiburg, Germany
February 2009

Wolfram Burgard

Preface

Models of the environment are needed for a wide range of robotic applications including search and rescue, transportation tasks, or automated vacuum cleaning. Learning maps has therefore been a major research topic in the robotics community over the last decades. Robots that are able to reliably acquire an accurate model of their environment on their own are regarded as fulfilling a major precondition of truly autonomous agents. To autonomously solve the map learning problem, a robot has to address mapping, localization, and path planning at the same time. In general, these three tasks cannot be decoupled and solved independently. Map learning is thus referred to as the simultaneous planning, localization, and mapping problem. Because of the coupling between these tasks, this is a complex problem. It can become even more complex when there are dynamic changes in the environment or several robots are being used together to solve the problem.

This book presents solutions to various aspects of the autonomous map learning problem. The book is separated into two parts. In the first part, we assume the position of the robot to be known. This assumption does not hold in the real world, however, it makes life easier and allows us to better concentrate on certain aspects of the exploration problem such as coordinating a team of robots. We describe how to achieve appropriate collaboration among exploring robots so that they efficiently solve their joint task. We furthermore provide a technique to learn and make use of background knowledge about typical spatial structures when exploring an environment as a team.

In the second part, we relax the assumption that the pose of the robot is known. To deal with the uncertainty in the pose of a robot, we present an efficient solution to the simultaneous localization and mapping problem. The difficulty in this context is to build a map while at the same time localizing the robot in this map. The presented approach maintains a joint posterior about the trajectory of the robot and the model of the environment. It produces accurate maps in an efficient and robust way. After addressing step-by-step the different problems in the context of active map learning, we integrate the main techniques into a single system. We present an integrated approach

that simultaneously deals with mapping, localization, and path planning. It seeks to minimize the uncertainty in the map and in the trajectory estimate based on the expected information gain of future actions. It takes into account potential observation sequences to estimate the uncertainty reduction in the world model when carrying out a specific action. Additionally, we focus on mapping and localization in non-static environments. The approach allows a robot to consider different spatial configurations of the environment and in this way makes the pose estimate more robust and accurate in non-static worlds.

In sum, the contributions of this book are solutions to various problems of the autonomous map learning problem including uncertainty-driven exploration, SLAM, active loop closing, coordination of multiple robots, learning and incorporating background knowledge, and dealing with dynamic environments.

A lot of the work presented in this book has been done in collaboration with other researchers. It was a pleasure for me to work with all the wonderful people in the AIS lab in Freiburg. First of all, I thank Wolfram Burgard for his tremendous support, his inspiration, and for providing a creative atmosphere. My thanks to my friends and colleagues for the great time in the lab, especially to Maren Bennewitz, Giorgio Grisetti, Dirk Hähnel, Óscar Martínez Mozos, Patrick Pfaff, Christian Plagemann, and Axel Rottmann for the great collaboration on the topics addressed in this book. It was a pleasure to work with all these people and to benefit from their knowledge. My thanks also to Mark Moors and Frank Schneider for the collaboration on multi-robot exploration. Special thanks to Nick Roy and Mike Montemerlo who did a great job in developing and maintaining the Carnegie Mellon Robot Navigation Toolkit. It was a pleasure for me to work together with all of them.

Additionally, I thank several people, who published robot datasets and in this way helped to make mapping approaches more robust and more easily comparable. In this context, I would like to thank Patrick Beeson, Mike Bosse, Udo Frese, Steffen Gutmann, Dirk Hähnel, Andrew Howard, and Nick Roy.

Freiburg, Germany
December 2008

Cyrill Stachniss

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Notation

Throughout this book, we make use of the following notation:

variable	description
x_t	pose of the robot at time step t . This pose is a three dimensional vector containing the x, y -position and the orientation θ of the vehicle
$x_{1:t}$	sequence of poses of the robot from time step 1 to time step t
z_t	sensor observation obtained at time step t
u_t	odometry information describing the movement from x_t to x_{t+1}
a	action or motion command
w	importance weight
$w_t^{[i]}$	importance weight of the i -th particle at time step t
m	grid map
c	grid cell
r	resolution of a grid map. Each cell covers an area of r by r .
\mathcal{G}	topological map
$E[\cdot]$	expectation
$\mathcal{N}(\mu, \Sigma)$	Gaussian with mean μ and covariance Σ
H	entropy
I	information gain
U	utility function
V	cost function
η	normalizer, typically resulting from Bayes' rule
N_{eff}	effective number of particles

Introduction

Models of the environment are needed for a wide range of robotic applications, from search and rescue to automated vacuum cleaning. Learning maps has therefore been a major research focus in the robotics community over the last decades.

In general, learning maps with single-robot systems requires the solution of three tasks, which are *mapping*, *localization*, and *path planning*. Mapping is the problem of integrating the information gathered with the robot's sensors into a given representation. It can be described by the question "What does the world look like?" Central aspects in mapping are the representation of the environment and the interpretation of sensor data. In contrast to this, localization is the problem of estimating the pose of the robot relative to a map. In other words, the robot has to answer the question, "Where am I?" Typically, one distinguishes between pose tracking, where the initial pose of the vehicle is known, and global localization, in which no a priori knowledge about the starting position is given. Finally, the path planning or motion control problem involves the question of how to efficiently guide a vehicle to a desired location or along a trajectory. Expressed as a simple question, this problem can be described as, "How can I reach a given location?"

Unfortunately, these three tasks cannot be solved independently of each other. Before a robot can answer the question of what the environment looks like given a set of observations, it needs to know from which locations these observations have been made. At the same time, it is hard to estimate the current position of a vehicle without a map. Planning a path to a goal location is also tightly coupled with the knowledge of what the environment looks like as well as with the information about the current pose of the robot.

The diagram in Figure 1.1 depicts the mapping, localization, and path planning tasks as well as the combined problems in the overlapping areas. *Simultaneous localization and mapping* (SLAM) is the problem of building a map while at the same time localizing the robot within that map. One cannot decouple both tasks and solve them independently. Therefore, SLAM is often referred to as a chicken or egg problem: A good map is needed for localization

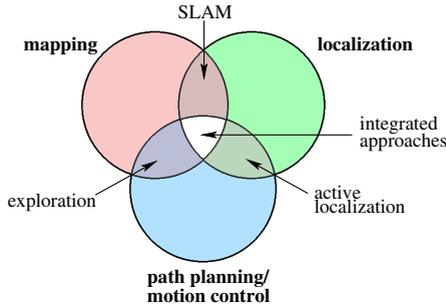


Fig. 1.1. Tasks that need to be solved by a robot in order to acquire accurate models of the environment. The overlapping areas represent combinations of the mapping, localization, and path planning tasks [94].

while an accurate pose estimate is needed to build a map. *Active localization* seeks to guide the robot to locations within the map to improve the pose estimate. In contrast to this, *exploration* approaches assume accurate pose information and focus on guiding the robot efficiently through the environment in order to build a map. The center area of the diagram represents the so-called *integrated approaches* which address mapping, localization, and path planning simultaneously. The integrated approaches are also called solutions to the *simultaneous planning, localization, and mapping* (SPLAM) problem. A solution to the SPLAM problem enables a mobile robot to acquire sensor data by autonomously moving through its environment while at the same time building a map. Whenever the robot is moving, it considers actions to improve its localization, to acquire information about unknown terrain, and to improve its map model by revisiting areas it is uncertain about. In the end, the robot is assumed to have learned an accurate model of the whole environment as well as determined its own pose relative to this model.

Several researchers focus on different aspects of these problems. This is done using single robot systems as well as teams of robots. The use of multiple robots has several advantages over single robot systems. Cooperating robots have the potential to accomplish a task faster than a single one. Furthermore, teams of robots can be expected to be more fault-tolerant than a single robot. However, when robots operate in teams, there is the risk of possible interference between them. The more robots that are used in the same environment, the more time each robot may spend on detours in order to avoid collisions with other members of the team. In most approaches, the performance of the team is measured in terms of the overall time needed to learn a map. This means that the robots need to be distributed over the environment in order to avoid redundant work and to reduce the risk of interference. A team of robots makes finding efficient solutions to problems like exploration more complex, since more agents are involved and so more decisions need to be made.

It is worth mentioning that all these problems become even more complex in the case where the environment changes over time. Most mapping techniques assume that the environment is static and does not change over time. This, however, is an unrealistic assumption, since most places where robots are used are populated by humans. Changes are often caused by people walking through the environment, by open and closed doors, or even by moved furniture. One possibility to deal with dynamic aspects is to filter them out and to map the static objects only. More challenging, however, is the problem of integrating the information about changes into the map and utilizing such knowledge in other robotic applications. This can enable a mobile robot to more efficiently execute its tasks. For example, one can expect a robot to more robustly localize itself in case where it knows about the typical configurations of the non-static aspects in its surroundings.

In summary, the key problems in the context of map learning are the questions of

- where to guide a robot during autonomous exploration,
- how to deal with noise in the pose estimate and in the observations,
- how to deal with the uncertainty in the robot's world model and how to interpret the sensor data,
- how to model changes in the environment over time, and
- how to efficiently coordinate a team of mobile robots.

The contributions presented in this book are solutions to different aspects of the map learning problem which explicitly consider these five aspects. We present approaches to autonomous exploration that take into account the uncertainty in the world model of the robot. We minimize this uncertainty by reasoning about possible actions to be carried out and their expected reward. We furthermore describe how to achieve good collaboration among a team of robots so that they efficiently solve an exploration task. Our approach effectively distributes the robots over the environment and in this way avoids redundant work and reduces the risk of interference between vehicles. As a result, the overall time needed to complete the exploration mission is reduced. To deal with the uncertainty in the pose of a robot, we present a highly accurate technique to solve the SLAM problem. Our approach maintains a joint posterior about the trajectory of the robot and the map model. It produces highly accurate maps in an efficient and robust way. In this book, we address step-by-step the problems in the context of map learning and integrate different solutions into a single system. We provide an integrated approach that simultaneously deals with mapping, localization, and path planning. It seeks to minimize the uncertainty in the map and trajectory estimate based on the expected information gain of future actions. It takes into account potential observation sequences to estimate the uncertainty reduction in the world model when carrying out a specific action. Additionally, we focus on mapping and localization in non-static environments. Our approach allows the robot

to consider different spatial configurations of the environment and in this way makes the pose estimate more robust and accurate in non-static worlds.

This book is organized as follows. First, we introduce the particle filtering technique and the ideas of grid maps. The first part of this book concentrates on single- and multi-robot exploration given the poses of the robots are known while they move through the environment.

Chapter 3 addresses the problem of decision-theoretic, autonomous exploration with a single vehicle. We consider a sensor which is affected by noise and investigate a technique to steer a robot through the environment in order to reduce the uncertainty in the map model.

In Chapter 4, we explore how to coordinate a team of robots in order to achieve effective collaboration and to avoid redundant work. The presented approach is extended in Chapter 5 so that background information about the structure of the environment is integrated into the coordination procedure. The knowledge about different structures is learned by the mobile robots from sensor data.

In the second part of this book, we relax the assumption of known poses and consider the uncertainty in the pose of a mobile robot. We present in Chapter 6 an efficient solution to the SLAM problem. It allows us to learn highly accurate grid maps while the pose information of the robot is affected by noise. Our technique maintains the joint posterior about the map and the trajectory of the robot using a particle filter. Chapter 7 describes a system to detect and to actively close loops during exploration. With this technique, we are not optimizing the pose estimation procedure but are planning appropriate trajectories for the mobile robot. The revisiting of known locations from time to time allows the robot to reduce the uncertainty in its pose. As a result, the obtained map is better aligned and shows less inconsistencies.

Actively revisiting known areas during SLAM offers not only the possibility to relocalize a vehicle, it also introduces the risk of becoming overly confident especially in the context of nested loops. To cope with this limitation, we present in Chapter 8 an approach for recovering the particle diversity after closing loops. This allows the robot to stay an arbitrary period of time within a loop without depleting important state hypotheses.

In Chapter 9, we present a decision-theoretic approach to exploration with respect to the uncertainty in the map and the pose estimate of the robot. The presented algorithm integrates different techniques introduced in the preceding chapters. It simultaneously addresses mapping, localization, and planning. As a result, our approach enables a real mobile robot to autonomously learn a model of the environment with low uncertainty even if its pose estimates are affected by noise.

Finally, Chapter 10 addresses the problem of mapping and localization in non-static environments. By explicitly modeling the different states the environment is observed in, the robot is able to more robustly localize itself in a non-static world.